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CLAN STRUCTURE IN RAPIDITY INTERVALS

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ABSTRACT

We present a cascading model for a single jet, inspired to QCD and to the phenomenological analysis of multiplicity distributions. The model, describing as it does a two dimensional evolution in virtuality and rapidity, allows analytical predictions for clan analysis parameters to be made.

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1. Introduction

Since its proposal[1], clan analysis has been widely used experimentally in order to interpret the negative binomial (NB) regularity and as a tool to classify different reactions[2]. It is especially important to stress the presence of the NB regularity when the multiplicity distributions (MD) are studied in intervals of rapidity, where conservation laws don't play a dominant role and therefore an understanding of the dynamics is most interesting and desirable.

Recently clan analysis has been extended in three directions: the regularity is better satisfied in the case of single jets, as is suggested by Monte Carlo studies[3] and experimentally by the observation that separating the 2-, 3- and 4-jet sample of events restores the regularity that was violated (in the shape of the multiplicity distribution, but not in the clan structure parameters) in the full sample of events[4]. Secondly, one should remember that, in going from the hadronic level to the partonic one via generalized local parton-hadron duality[5], the NB regularity is better satisfied. This suggests that an explanation of the regularity should be sought at partonic level. The third extension comes by recognizing that the clan interpretation of the negative binomial distribution (NBD) is actually more general than the NBD itself, and leads to the idea that the emission of partons in a single jet is a two-step process[6].

The difficulty of analytical calculations in QCD, and a guess on the real nature of clans in multiparticle production (they might very well be true physical objects) has prompted us to develop an analytical parton shower model for a single jet inspired to QCD and at the same time close to the phenomenological observations.

In this paper we first discuss some properties of two-step processes in general, and once this framework has been established, we describe the model and its results.

2. Two-step processes

Assume that the parton production process involves two independent steps: in the first step N objects ($N = 0, 1 \ldots$), (which we call *clans*, and the clans known from the NBD are now a particular case), are produced with probability p_N and generating function $f(z)$ (in all quantities in this Section a dependence on the jet energy is understood). In the second step, each clan produces partons with probability q_{n_i} (with i labeling the clans: $i = 1, 2 \ldots N$) and generating function $g(z)$. Since clans are identified by the final partons they generate, requiring that each clan produce at least one parton (i.e, $q_0 = 0$) makes the number of clans unambiguously defined. Notice that we do not assume that all clans are created equal: the multiplicity distribution (MD) of an individual clan can depend on a set of parameters ξ as $\tilde{q}_{n_i}(\xi)$: if the value of ξ for a clan is independent of that of other clans and on the number of clans, one can define an *average clan*:

$$q_{n_i} = \int \tilde{q}_{n_i}(\xi)\phi(\xi)d\xi \; , \tag{1}$$

where $\phi(\xi)$ is the p.d.f. for producing a clan with that values of the parameters. Indeed this property will be used in Sec. 4.

With these assumptions, the final partons MD, P_n, has generating function

$$F(z) \equiv \sum_{n=0}^{\infty} z^n P_n = f\,(\,g(z)\,)\;\; . \tag{2}$$

If we turn now to look at the production in intervals of phase space, it is clear that the only dependence on the particular interval belongs to the second step, which is the step that produces the final partons. Choosing for definiteness to look at a rapidity interval which will be denoted Δy, we find that each clan produces partons according to

$$g(z, \Delta y) = \sum_{n=0}^{\infty} z^n q_n(\Delta y) \; , \qquad\qquad q_0(\Delta y) \neq 0 \tag{3}$$

where the fact that it is possible that a clan produces zero partons inside Δy has been emphasized (see also Fig. 1). Obviously the final distribution is then given by

$$F(z, \Delta y) = f\,(\,g(z, \Delta y)\,)\,. \tag{4}$$

The parameters of interest in this paper are then the average number of clans, $\bar{N}$, and the average number of partons per clan, $\bar{n}_c^{(0)}$:

$$\bar{N} \equiv \left.\frac{df(z)}{dz}\right|_{z=1} \qquad\qquad \bar{n}_c^{(0)}(\Delta y) \equiv \left.\frac{dg(z,\Delta y)}{dz}\right|_{z=1} = \frac{\bar{n}(\Delta y)}{\bar{N}} \tag{5}$$

where $\bar{n}(\Delta y)$ is the average number of final partons from Eq. (4).

2.1. Binomial convolution

$f(z)$ and $g(z, \Delta y)$ can be redefined so that only clans which produce at least one parton in the interval Δy are considered: one simply subtracts $q_0(\Delta y)$ and rescales the distribution correspondingly:

$$g_1(z, \Delta y) = \frac{g(z, \Delta y) - q_0(\Delta y)}{1 - q_0(\Delta y)} \; . \tag{6}$$

This implies then

$$f_1(z) = f\Big([1 - q_0(\Delta y)]z + q_0(\Delta y)\Big) \; . \tag{7}$$

It can be shown that this is equivalent to the condition that the probability that m of the N produced clans generate at least one parton in Δy is given by a binomial distribution of parameter $[1 - q_0(\Delta y)]$. In particular, the average number of clans contributing to the interval Δy is in general

$$\bar{N}(\Delta y) = [1 - q_0(\Delta y)]\bar{N} \tag{8}$$

and therefore one obtains

$$\bar{n}_c^{(1)}(\Delta y) = \frac{\bar{n}(\Delta y)}{\bar{N}(\Delta y)} = \frac{\bar{n}_c^{(0)}(\Delta y)}{[1 - q_0(\Delta y)]} \,. \tag{9}$$

Example 1A. Suppose that $f(z)$ is a Poisson distribution:

$$f(z) = e^{\bar{N}(z-1)}; \tag{10}$$

then the redefined distribution is still a Poisson distribution with parameter given by Eq. (8):

$$F(z, \Delta y) = \exp\{\bar{N}(\Delta y)[g_1(z, \Delta y) - 1]\} \,. \tag{11}$$

Example 1B. Consider the case in which $f(z)$ is a shifted Poisson distribution:

$$f(z) = ze^{\bar{N}(z-1)}; \tag{12}$$

(such a case will be encountered in Sec. 3). Then the result is more complex:

$$F(z, \Delta y) = \Big\{[1 - q_0(\Delta y)]g_1(z, \Delta y) + q_0(\Delta y)\Big\}e^{\bar{N}(\Delta y)[g_1(z,\Delta y)-1]} \,. \tag{13}$$

2.2. Compound Poisson distribution

Alternatively, one can try and redefine the generating functions so that the final MD is a compound Poisson distribution; this is an implicit assumption made when one makes a fit with a NBD. We want to solve:

$$F(z, \Delta y) = \exp\{\lambda(\Delta y)[g_2(z, \Delta y) - 1]\} \tag{14}$$

$$g_2(0, \Delta y) = 0 \tag{15}$$

for $\lambda(\Delta y)$ and $g_2(z, \Delta y)$ and we obtain:

$$\lambda(\Delta y) = -\log[F(0, \Delta y)] \tag{16}$$

$$g_2(z, \Delta y) = 1 + \frac{\log[F(z, \Delta y)]}{\lambda(\Delta y)} \,. \tag{17}$$

Here $g_2(z, \Delta y)$ is a true probability generating function ($d^n g_2/dz^n \geq 0 \; \forall n$) if and only if all the combinants of the distribution F are positive. In other words, in some cases it may not be possible to carry out this redefinition. In case it is, the parameters of interest are $\lambda(\Delta y)$ as derived above and

$$\bar{n}_c(\Delta y) = \frac{\bar{n}(\Delta y)}{\lambda(\Delta y)} = \frac{\bar{N}\,\bar{n}_c^{(0)}(\Delta y)}{\lambda(\Delta y)}\,. \tag{18}$$

Example 2A. For a Poissonian $f(z)$ we find the same result as in example *1A*:

$$\begin{aligned} \lambda(\Delta y) &= [1 - q_0(\Delta y)]\bar{N} && (19)\\ g_2(z,\Delta y) &= \frac{g(z,\Delta y) - q_0(\Delta y)}{1 - q_0(\Delta y)} && (20) \end{aligned}$$

so that the two transformations coincide. In particular, $\lambda(\Delta y)$ is equal to the average number of clans in the interval Δy.

Example 2B. For a shifted Poisson, on the other hand, they do not coincide:

$$\begin{aligned} \lambda(\Delta y) &= [1 - q_0(\Delta y)]\bar{N} - \log[q_0(\Delta y)] && (21)\\ g_2(z,\Delta y) &= 1 + \frac{\bar{N}[g(z,\Delta y) - 1] + \log[g(z,\Delta y)]}{\lambda(\Delta y)} && (22) \end{aligned}$$

and $\lambda(\Delta y)$ is not equal to $\bar{N}(\Delta y)$. In particular it should be noted that in full phase space (fps), $\bar{N}(\text{fps}) = \bar{N}$ but $\lambda(\text{fps})$ goes to infinity.

3. The GSPS model

The GSPS model is introduced in order to solve analytically a two dimensional parton evolution in a single jet[6,7]. It describes an ancestor parton, which degrades in virtuality, and which we follow in rapidity, emitting clans of different virtuality and rapidity; each clan emits partons in a two dimensional cascade process, see Fig. 1.

3.1. Step One

In Fig. 1 the first step is shown with thick lines. Because clans are by assumption emitted independently in a cascade fashion, we take the two branches in each splitting of the ancestor to be independently regulated by the same splitting function, for which a form inspired by QCD, suitably normalized by a Sudakov form factor term, has been chosen:

$$p_A(Q_0|Q)dQ_0 = p_0^A(Q)C_A(Q)dQ_0 = C_A(Q)\frac{d}{dQ_0}\left(\frac{1}{C_A(Q_0)}\right)dQ_0 = d\left(\frac{\log Q_0}{\log Q}\right)^A\,. \tag{23}$$

The parameter $A > 0$ thus controls the branching of the ancestor: to a larger value of A corresponds more branching.

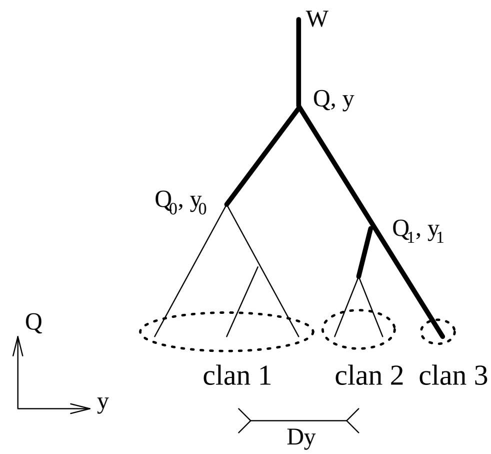

Fig. 1. Schematic representation of the GSPS model. Thick lines indicate clans creation (step 1) and thin lines indicate cascading into final partons (step 2). Notice that only clan 1 and 2 produce partons in the interval Dy shown.

In order to apply this factorization, we have to allow for local violations of the energy-momentum conservation law, still requiring its global validity, i.e., offspring partons of virtualities Q_i can fluctuate according to:

$$Q_0 + Q_1 \not\leq Q \quad , \quad 1\ \text{GeV}\ \leq Q_i \leq Q \quad [\text{i=0,1}]\ . \tag{24}$$

Of course, constraints on the energy fraction carried away by daughter partons are also no longer valid, i.e., $z_0 + z_1 \neq 1$, and kinematic bounds in rapidity are loosened:

$$|y_i - y| \leq \log \frac{Q}{Q_i} \qquad [\text{i=0,1}]\ . \tag{25}$$

This new condition modifies also the splitting kernel in z which now is decoupled:

$$P(z_0, z_1) dz_0 dz_1 \propto \frac{dz_0}{z_0} \frac{dz_1}{z_1}\ . \tag{26}$$

Notice that, for each branch, we have taken the singular part of the Altarelli-Parisi kernel.

Finally, let us mention that the rapidity of the first splitting is obtained from the energy W and the virtuality Q by the simple kinematic relationship:

$$y = \tanh^{-1} \sqrt{1 - Q^2/W^2}. \tag{27}$$

3.2. Step Two

At this point, clans have been emitted with definite initial virtualities and rapidities. The simplest and most economic assumption is now that inside each clan a cascade process develops in a way very similar to what has been described for the first step: each parton branches into two independent partons. The same form of splitting functions will be applied as in Sec. 3.1, but with a different parameter:

$$p_a(Q_0|Q)dQ_0 = d\left(\frac{\log Q_0}{\log Q}\right)^a \quad ; \tag{28}$$

to a larger value of a corresponds then a larger number of branchings, and therefore of final partons. The rapidity part is treated according to Eq. (25) and (26).

Finally, one should note that, if energy conservation were strictly enforced, a parton with virtuality less than 2 GeV could not split. Because of Eq. (24) this is no longer necessary, but we keep it nonetheless as our cut-off procedure.

4. The structure of the calculation

The calculation has been described in detail in [7], here we will only outline its structure.

Since, according to the model, clans of given virtuality and rapidity are produced, one must first calculate the generating function for the MD of a clan with initial virtuality Q and rapidity y to produce partons in the interval Δy, which we call $g(z, \Delta y, Q, y)$. It would correspond to Eq. (3) in Sect. 2. It can be done based on step two of the model only.

We then calculate the probability to emit a clan with initial virtuality Q and initial rapidity y, which we call $\phi(Q, y)$. This can be done based on step one of the model only.

Finally one defines the generating function for the MD of an average clan similarly to what is done in Eq. (1):

$$g(z, \Delta y) = \int_1^W dQ \int dy\, g(z, \Delta y, Q, y)\phi(Q, y). \tag{29}$$

This distribution is then to be convoluted with the generating function for the number of clans, $f(z, W)$, which depends of course only on step one of the model.

Example. It is perhaps best to illustrate this procedure by an example, which for simplicity only regards full phase space[8]. Let us examine a pure birth model for a single clan: the MD is a shifted geometrical distribution

$$g(z, \text{fps}, Q, y) = \frac{z}{z - \nu(Q)(z-1)} \tag{30}$$

where the average value $\nu(Q)$ is assumed (for the sake of this example only: in the GSPS model its explicit form can be calculated analytically) to be a specific function of the clan virtuality Q:

$$\phi(Q)dQ \propto \frac{d\nu}{\nu}. \tag{31}$$

The MD of an average clan is given by

$$g(z) = \frac{1}{\log \nu} \int_1^{\nu} \frac{z}{z - \nu'(z-1)} \frac{d\nu'}{\nu'} = \frac{\log(1-bz)}{\log(1-b)} \tag{32}$$

where $b = 1 - 1/\nu$. It is a logarithmic distribution which, when convoluted with a Poisson distribution for the first step, gives a NBD. It should be pointed out that this example does not apply directly to the GSPS: as shown in the next section, the MD of an average clan is not a logarithmic distribution (but resembles it); the MD for clans is not a Poisson distribution, but a shifted Poisson distribution.

5. Results and discussion

The model can be solved analytically under a few mild mathematical approximations: the full solution in rapidity intervals is however far from being simple and compact. The results are therefore presented in graphical form, but in order to get the general flavour it is interesting to quote the analytical result in full phase space: the MD of final clans is given by

$$f(z, W) = z \exp\Big\{[\bar{N}(W) - 1][z - 1]\Big\} ; \tag{33}$$

because at least one parton (the ancestor) is always present in the cascade, the total distribution of clans turns out to be a shifted Poisson distribution. The distribution inside an average clan is given by

$$g(z, W) = \frac{1}{\bar{N}(W)} \left\{ z + \frac{A}{a} u_a(W) - \frac{A}{a} \log\left[z + (1-z)\exp\{u_a(W)\}\right] \right\} . \tag{34}$$

where

$$u_a(W) \equiv \int_2^W p_a(Q|Q) dQ = a \log\left(\frac{\log W}{\log 2}\right) . \tag{35}$$

The final partons distribution is then obtained by inserting Eq. (34) into Eq. (33) according to Eq. (4).

The clan parameters can then be calculated to be

$$\bar{N}(W) = 1 + A \log\left(\frac{\log W}{\log 2}\right) \tag{36}$$

and

$$\bar{n}_c(W) = \frac{1}{\bar{N}(W)} \left\{ 1 - \frac{A}{a}\left[1 - \left(\frac{\log W}{\log 2}\right)^a\right] \right\} . \tag{37}$$

As expected, the average number of clans depends only on the parameter A which regulates the first step of the emission process; the average number of partons in an *average* clan depends on both parameters (whereas the distribution in a single clan

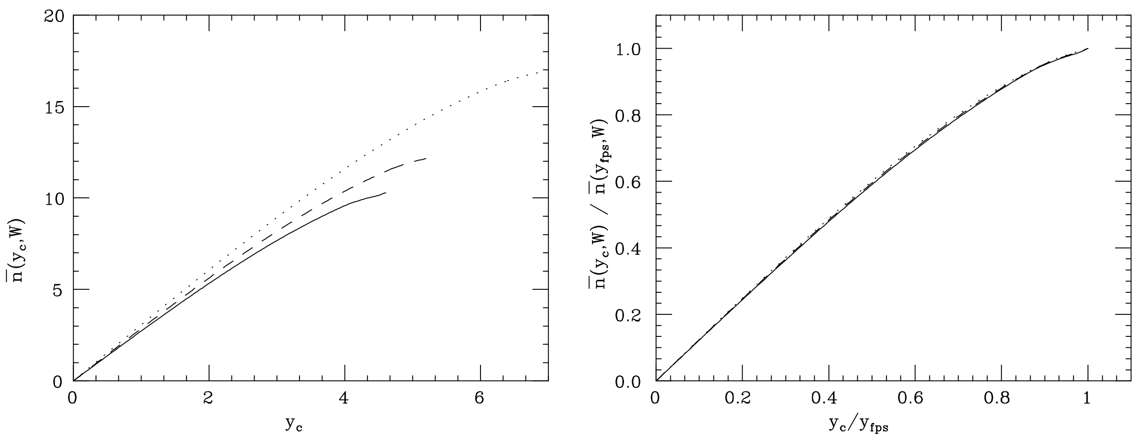

Fig. 2. *Left:* Average number of partons in the shower, $\bar{n}(y_c, W)$, as a function of the width of the rapidity interval y_c obtained analytically in the GSPS model with $A = 2$, $a = 1$ at different maximum allowed virtualities $W = 50$ GeV (solid line), 100 GeV (dashed line) and 500 GeV (dotted line). *Right:* The same quantity is plotted normalized to the value in full phase space as a function of the rescaled rapidity interval.

of definite virtuality depends of course only on a). It should be noticed that in the limit of high initial virtuality (energy), $W \to \infty$, the dependence of $\bar{n}_c(W)$ on A disappears.

The solution in rapidity intervals is, as explained in the previous section, a convolution of Eq. (33) with the MD of an average clan; however, since we are here interested only on clan parameters properties, we will calculate only these, and not the full distribution.

The actual calculations in rapidity intervals have been carried out for $A = 2$ and $a = 1$, values which avoid nasty inessential calculations and make possible the analytical solution of the model without hurting its logic.

In Fig. 2 we plot the average number of final partons in both the standard form (on the left) and in a rescaled form (to the right), showing an interesting scaling behaviour with energy. This scaling in W is found to depend on the parameter a, as different values of a give different scaling curves, differently from the scaling found in[6] for $\bar{N}(\Delta y, W)/\bar{N}(\mathrm{fps}, W)$ which by its own definition is independent of the mechanism at work inside clans.

In Fig. 3 we plot the parameter $\lambda(\Delta y, W)$, defined by requiring that full distribution be a compound Poisson distribution, and the corresponding average number of partons per clan. Remember that is the parameter λ which should be compared, *in rapidity intervals not too close to full phase space*, with the average number of clans as obtained from NBD fits. One can clearly see in the figure the linear rise of $\lambda(\Delta y, W)$ as Δy grows, and the slow decrease with energy at fixed Δy. The result for $\bar{n}_c(\Delta y, W)$ is reasonable but not fully satisfactory: in the limit $\Delta y \to 0$, $\bar{n}_c(\Delta y, W)$ points to constant values, which differ at different energies, in contrast with the ex-

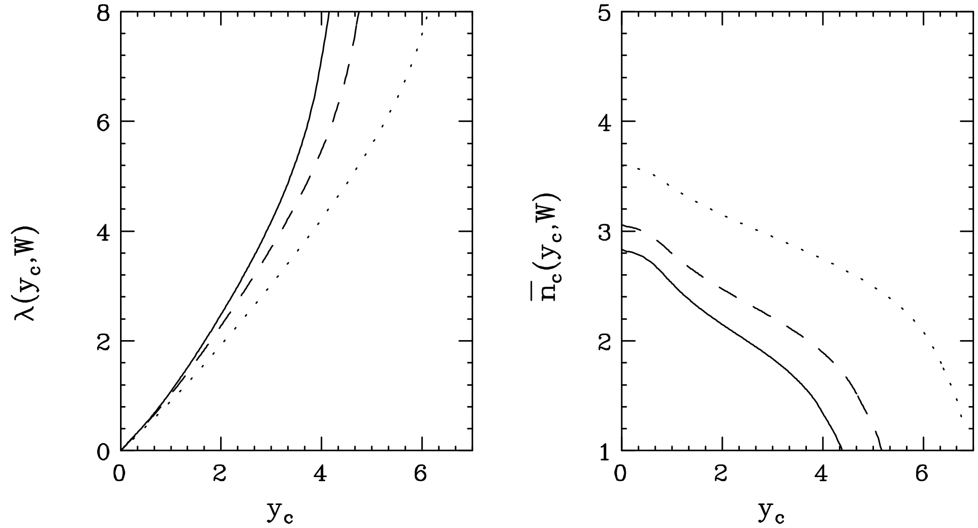

Fig. 3. Results of the GSPS model for the parameters of clan analysis as a function of the width of the rapidity interval y_c at different maximum allowed virtualities $W = 50$ GeV (solid line), 100 GeV (dashed line) and 500 GeV (dotted line). Analytical solution with $A = 2$ and $a = 1$.

pected W-independent value $\bar{n}_c(0, W) = 1$. The explanation for this anomaly lies in the approximation we had to use in the analytical calculations, which fail in the very small intervals $\Delta y < 1$. Indeed a Monte Carlo version of the model shows[7] that the slope of $\lambda(\Delta y, W)$ when $\Delta y \to 0$ matches that of $\bar{n}(\Delta y, W)$ in Fig. 2, so that actually one finds $\bar{n}_c(0, W) = 1$.

6. Conclusions

We have illustrated the GSPS model and its results; it is a parton shower model which was built from QCD-inspired splitting functions in virtuality and in rapidity, with Sudakov form factor normalization, to which the phenomenologically established idea of clans was added by allowing at each step in the cascading local violations of the energy-momentum conservation law (which is recovered globally in a statistical sense). The model has important predictive power in regions not presently accessible to full perturbative QCD; the results on clan analysis are consistent with what we know of single gluon jets disentangled using a jet finding algorithm from the JETSET Monte Carlo program and analyzed at parton level by assuming generalized local parton-hadron duality[3]. These predictions can (and hopefully will) be tested in the near future as clean samples of single gluon and quark jets of different energies have been separated at LEP[9]; some preliminary data are available in full phase space only, an analysis of which is under way[10].

7. Acknowledgements

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